EUROPHOTONICS-POESII MASTER COURSE

PROPOSAL FOR A MASTER THESIS

Dates: April 1st, 2016 – September 30th, 2016

Laboratory: Quantum Information Theory, ICFO-The Institute of Photonic Sciences
City, Country: Castelldefels (Barcelona), Spain

Title of the master thesis: Analysing Many-Particle Quantum Systems with Semidefinite Programming Relaxations

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Summary of the subject (maximum 1 page) :

Many relevant problems in quantum physics can be understood as the minimization of a polynomial over non-commuting variables subject to some positivity constraints. Examples of these problems are the computation of the maximal violation of Bell inequalities (foundations of quantum physics), the amount of certifiable quantum randomness (quantum information theory), or the computation of the ground state-energy of many-body states (condensed matter physics) or of molecules (quantum chemistry).

Recently, a general formalism to approach these optimization problems has been derived in terms of an infinite hierarchy of semi-definite programming (SDP) relaxations. Each step in the hierarchy provides a lower bound to the minimum and the sequence of lower bounds converges to the searched solution. This hierarchy can be understood as the non-commutative analogue of existing optimization methods derived for commuting classical variables by Lasserre and Parrilo. The hierarchy was first introduced in the context of foundations of quantum physics and quantum information theory, and it has been proven to be the best existing method to solve some optimization problems in these two fields. Later, a general mathematical framework for the hierarchy was derived, where it clearly emerged its potential application to other fields, such as many-body physics. One of the main advantages of the hierarchy is that each of its steps defines an SDP problem, making its numerical calculation feasible.

The main objective of the thesis is to apply the hierarchy of SDP relaxations to many-body systems. The paradigmatic problem we plan to consider is the computation of the ground state

energy of a many-body system. It is well known that this is a computationally demanding problem. The focus will be on interacting bosonic and fermionic systems in a chain and in a 2D lattice. These are very interesting models that have often been proposed as a mechanism to understand high-temperature super-conductivity.

We will first consider exactly solvable models and compare the obtained bounds with the known analytical solution. Then, we will move to non-solvable models for which different numerical ansatzs have been proposed. We will compare the upper bounds obtained via these ansatzs with the lower bounds derived from the SDP hierarchy. By studying all these different problems, we hope to identify physically relevant situations in which SDP relaxations beat any of the existing numerical methods to establish lower bounds to the ground-state energy.

Keywords : ground-state energy, phase transition, second quantization, polynomial optimization problem, semi-definite programming, convex relaxation

Additional information :
* Required skills : working knowledge of Python, numerical methods
* Miscellaneous :